Name: ______________________________ Date: ____________

PC: Linear Programming Ms. Loughran

Do Now:

1. Graph the solution of:

$x \geq 0$

$y \geq 0$

$x + 2y \leq 4$

$x - y \leq 1$

Many applications in business and economics involve a process called **optimization,** in which you have to find the minimum or maximum value of a quantity. We are going to use an optimization strategy called **linear programming**. A two-dimensional linear programming consists of a linear **objective function** and a system of linear inequalities called **constraints.** The objective function gives the quantity that is to be maximized or minimized and the constraints determine the set of feasible solutions.

If a linear programming problem has a solution, it must occur at a vertex of the feasible solutions. If there is more than one solution, at least one of them must occur at such a vertex. In either case, the value of the objective function is unique.

1. Find the maximum value of

$z = 3x + 2y$

subject to the following constraints,

$x \geq 0$

$y \geq 0$

$x + 2y \leq 4$

$x - y \leq 1$

2. Find the maximum value of

$$z = 4x + 6y$$

subject to the following constraints,

$$x \geq 0$$

$$y \geq 0$$

$$-x + y \leq 11$$

$$x + y \leq 27$$

$$2x + 5y \leq 90$$

3. Find the minimum value of

$$z = 5x + 7y$$

subject to the following constraints,

$$x \geq 0$$

$$y \geq 0$$

$$2x + 3y \geq 6$$

$$3x - y \leq 15$$

$$-x + y \leq 4$$

$$2x + 5y \leq 27$$

4. Find the maximum value of

$$z = 5x + 7y$$

subject to the following constraints,

$x \geq 0$

$y \geq 0$

$2x + 3y \geq 6$

$3x - y \leq 15$

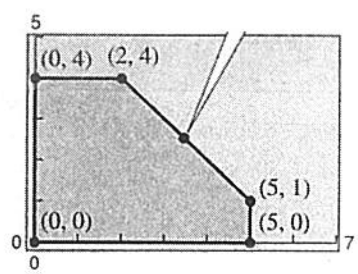
$-x + y \leq 4$

$2x + 5y \leq 27$

5. Find the maximum value of

$$z = 2x + 2y$$

given the following region corresponding to the system of constraints:



6. . Find the maximum value of

$$z = 4x + 2y$$

subject to the following constraints,

$$x \geq 0$$

$$y \geq 0$$

$$x + 2y \geq 4$$

$$3x + y \geq 7$$

$$-x + 2y \leq 7$$

7. A manufacturer wants to maximize the profit for two products. Product 1 yields a profit of $1.50 per unit and product 2 yields a profit of $2.00 per unit. Market tests and available resources have indicated the following constraints.

   1. The combined production level should not exceed 1200 units per month.
   2. The demand for product 2 is no more than half the demand for product 1.
   3. The production level of product 1 is less than or equal to 600 units plus three times the production level of product 2.

Solving a Linear Programming Problem

To solve a linear programming problem involving two variables by the graphical method, use the following steps.

1. Sketch the region corresponding to the system of constraints.
2. Find the vertices of the region.
3. Test the objective function at each of the vertices and select the values of the variables that optimize the objective function. For a bounded region, both a minimum and a maximum value will exist. For an unbounded region, if an optimal solution exists, it will occur at a vertex.

Practice (from textbook)

In Exercises 1–12, find the minimum and maximum values of the objective function, subject to the indicated constraints. (For each exercise, the graph of the region determined by the constraints is provided.)

1. Objective function:

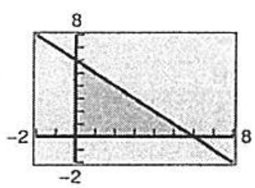
$z = 4x + 5y$

Constraints:

$x \geq 0$

$y \geq 0$

$x + y \leq 6$



2. Objective function:

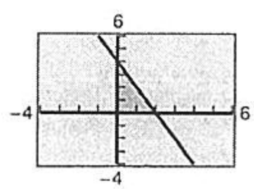
$z = 2x + 8y$

Constraints:

$x \geq 0$

$y \geq 0$

$2x + y \leq 4$



3. Objective function:

$z = 10x + 6y$

Constraints:
(See Exercise 1.)

4. Objective function:

$z = 7x + 3y$

Constraints:
(See Exercise 2.)

5. Objective function:

$z = 3x + 2y$

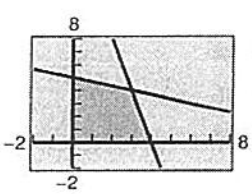
Constraints:

$x \geq 0$

$y \geq 0$

$x + 3y \leq 15$

$4x + y \leq 16$



6. Objective function:

$z = 4x + 3y$

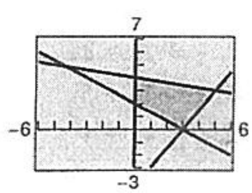
Constraints:

$x \geq 0$

$2x + 3y \geq 6$

$3x - 2y \leq 9$

$x + 5y \leq 20$



7. Objective function:

$z = 5x + 0.5y$

Constraints:
(See Exercise 5.)

8. Objective function:

$z = x + 6y$

Constraints:
(See Exercise 6.)

9. Objective function:

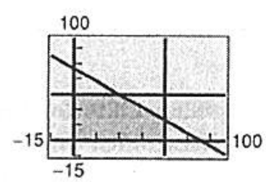
$z = 10x + 7y$

Constraints:

$0 \leq x \leq 60$

$0 \leq y \leq 45$

$5x + 6y \leq 420$



10. Objective function:

$z = 50x + 35y$

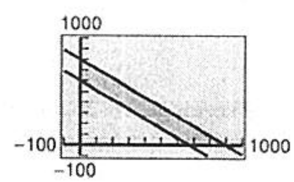
Constraints:

$x \geq 0$

$y \geq 0$

$8x + 9y \leq 7200$

$8x + 9y \geq 5400$



11. Objective function:

$z = 25x + 30y$

Constraints:
(See Exercise 9.)

12. Objective function:

$z = 16x + 18y$

Constraints:
(See Exercise 10.)

In Exercises 13–20, sketch the constraint region. Then find the minimum and maximum values of the objective function, subject to the constraints.

13. Objective function:

$z = 6x + 10y$

Constraints:

$x \geq 0$

$y \geq 0$

$2x + 5y \leq 10$

14. Objective function:

$z = 7x + 8y$

Constraints:

$x \geq 0$

$y \geq 0$

$x + \frac{1}{2}y \leq 4$

15. Objective function:

$z = 9x + 24y$

Constraints:
(See Exercise 13.)

16. Objective function:

$z = 7x + 2y$

Constraints:
(See Exercise 14.)

17. Objective function:

$z = 4x + 5y$

Constraints:

$$\begin{aligned} x &\geq 0 \\ y &\geq 0 \\ x + y &\geq 8 \\ 3x + 5y &\geq 30 \end{aligned}$$

18. Objective function:

$z = 4x + 5y$

Constraints:

$$\begin{aligned} x &\geq 0 \\ y &\geq 0 \\ 2x + 2y &\leq 10 \\ x + 2y &\leq 6 \end{aligned}$$

19. Objective function:

$z = 2x + 7y$

Constraints:
(See Exercise 17.)

20. Objective function:

$z = 2x - y$

Constraints:
(See Exercise 18.)

In Exercises 21–26, use a graphing utility to sketch the region determined by the constraints. Then find the minimum and maximum values of the objective function, subject to the constraints.

21. Objective function:

$z = 4x + y$

Constraints:

$$\begin{aligned} x &\geq 0 \\ y &\geq 0 \\ x + 2y &\leq 40 \\ 2x + 3y &\geq 72 \end{aligned}$$

22. Objective function:

$z = x$

Constraints:

$$\begin{aligned} x &\geq 0 \\ y &\geq 0 \\ 2x + 3y &\leq 60 \\ 2x + y &\leq 28 \\ 4x + y &\leq 48 \end{aligned}$$

23. Objective function:

$z = x + 4y$

Constraints:
(See Exercise 21.)

24. Objective function:

$z = y$

Constraints:
(See Exercise 22.)

25. Objective function:

$z = 2x + 3y$

Constraints:
(See Exercise 21.)

26. Objective function:

$z = 3x + 2y$

Constraints:
(See Exercise 22.)

Exploration In Exercises 27–30, (a) use a graphing utility to graph the region bounded by the following constraints.

$$\begin{aligned} 3x + y &\leq 15 \\ 4x + 3y &\leq 30 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$$

(b) Graph the objective function for the given maximum value of z in the same viewing rectangle as the graph of the constraints. (c) Use the graph to determine the feasible point or points that yield the maximum. Explain how you arrived at your answer.

	Objective Function	*Maximum*
27.	$z = 2x + y$	$z = 12$
28.	$z = 5x + y$	$z = 25$
29.	$z = x + y$	$z = 10$
30.	$z = 3x + y$	$z = 15$

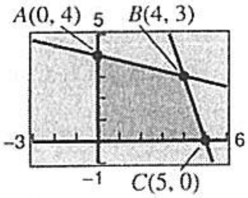
Exploration In Exercises 31–34, (a) use a graphing utility to graph the region bounded by the following constraints.

$$\begin{aligned} x + 4y &\leq 20 \\ x + y &\leq 8 \\ 3x + 2y &\leq 21 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$$

(b) Graph the objective function for the given maximum value of z in the same viewing rectangle as the graph of the constraints. (c) Use the graph to determine the feasible point or points that yield the maximum. Explain how you arrived at your answer.

	Objective Function	*Maximum*
31.	$z = x + 5y$	$z = 25$
32.	$z = 2x + 4y$	$z = 24$
33.	$z = 4x + 5y$	$z = 36$
34.	$z = 4x + y$	$z = 28$

Think About It In Exercises 35–38, find an objective function that has a maximum or minimum value at the indicated vertex of the constraint region shown below. (There are many correct answers.)



35. The maximum occurs at vertex A.

36. The maximum occurs at vertex B.

37. The maximum occurs at vertex C.

38. The minimum occurs at vertex C.

39. *Maximum Profit* A merchant plans to sell two models of compact disc players at costs of \$250 and \$400. The \$250 model yields a profit of \$45, and the \$400 model yields a profit of \$50. The merchant estimates that the total monthly demand will not exceed 250 units. The merchant does not want to invest more than \$70,000 in inventory for these products. Find the number of units of each model that should be stocked in order to maximize profit.

40. *Maximum Profit* A fruit grower has 150 acres of land available to raise two crops, A and B. It takes 1 day to trim an acre of crop A and 2 days to trim an acre of crop B, and there are 240 days per year available for trimming. It takes 0.3 day to pick an acre of crop A and 0.1 day to pick an acre of crop B, and there are 30 days available for picking. The profits are \$140 per acre for crop A and \$235 per acre for crop B. Find the number of acres of each fruit that should be planted to maximize profit.

41. *Minimum Cost* Two gasolines, type A (\$1.13 per gallon) and type B (\$1.28 per gallon), have octane ratings of 80 and 92, respectively. Determine the blend of minimum cost with an octane rating of at least 90. (*Hint:* Let x be the fraction of each gallon that is type A and let y be the fraction that is type B.)

42. *Maximum Revenue* An accounting firm has 900 hours of staff time and 100 hours of reviewing time available each week. The firm charges \$2000 for an audit and \$300 for a tax return. Each audit requires 100 hours of staff time and 10 hours of review time. Each tax return requires 12.5 hours of staff time and 2.5 hours of review time. What numbers of audits and tax returns will yield the maximum revenue?

43. *Maximum Revenue* The accounting firm in Exercise 42 lowers its charge for an audit to \$1000. What numbers of audits and tax returns will yield the maximum revenue?

44. *Maximum Profit* A manufacturer produces two models of bicycles. The amounts of time (in hours) required for assembling, painting, and packaging the two models are as follows.

	Model A	Model B
Assembling	2	2.5
Painting	4	1
Packaging	1	0.75

The total amounts of time available for assembling, painting, and packaging are 4000, 4800, and 1500 hours, respectively. The profits per unit are \$45 (model A) and \$50 (model B). How many of each model should be produced to maximize profit?

45. *Maximum Profit* A manufacturer produces two models of bicycles. The amounts of time (in hours) required for assembling, painting, and packaging the two models are as follows.

	Model A	Model B
Assembling	2.5	3
Painting	2	1
Packaging	0.75	1.25

The total amounts of time available for assembling, painting, and packaging are 4000, 2500, and 1500 hours, respectively. The profits per unit are \$50 (model A) and \$52 (model B). How many of each model should be produced to maximize profit?